Harrod’s Dynamics and the Kaldor-Thirlwall Export-led Growth

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Abstract

This paper investigates an open economy macrodynamic model on Harrodian lines. The analysis starts with a reconsideration of the post-Keynesian Thirlwall’s approach to growth in open economy. We cast doubt on the Thirlwall idea aimed at rendering the concept of Harrod’s foreign trade multiplier genuinely consistent with Harrod’s dynamics. Following a different procedure from Thirlwall’s approach, we start from Harrod’s dynamics in a closed economy and investigate what happens when the foreign trade is introduced. An important result of our model is that, according to Harrod’s intuition, there is less cyclical instability in the open economy than in the closed one where chaotic instability prevails. Dynamics in our model exhibits a limit cycle. Finally, the model shows that the possibility of an export-led growth strictly depends on the technical progress dynamics.

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1. Introduction

In recent years, accelerated processes of international integration have interested the world economy. The movement towards increasing liberalization in trade flows, FDI and financial transactions has not followed the lines postulated by neoclassical theory. Permanent current account imbalances in many countries, increasing disparities in per capita incomes between advanced and non advanced economies, regional divergences in growth rates, high levels of unemployment and persistent situations of stagnation especially among EU countries are all facts hard to explain in the neoclassical framework.

During the eighties and the nineties of the last century novel analytical efforts were implemented to shed new light on stylized facts associated with the evolution of the world economic integration.

On the one hand, endogenous growth models (EG) have focussed on the auto-propelling role of technological change and the accumulation of human capital in boosting long run growth (Romer, 1990; Aghion and Howitt, 1992). When these models are extended to open economy (Grossman and Helpman, 1991), world convergence paths are found to be dependent on whether and how technology has spread internationally.

On the other hand, new economic geography (NEG) models have stressed how the interaction between transport costs, increasing returns and market size could explain spatial agglomeration of economic activities according to a mechanism of cumulative causation (Krugman, 1991; Krugman and Venables, 1995). However, in a 1970 seminal article, Nicholas Kaldor had already offered a demand-based conceptual framework of regional growth in which the interaction between exports, increasing returns, and efficiency wages (in the Keynesian sense) could generate a cumulative development gap between advanced and non advanced areas. The Kaldor approach was formalized by Dixon and Thirlwall (1975) in accordance with the pioneering export-led growth model of the sixties (Beckerman, 1962; Lamfalussy, 1963). Successive works by Thirlwall have tried to consolidate definitively the contemporary analysis of open economy macrodynamics based on the Keynes-Harrod-Kaldor tradition (Thirlwall, 1979; Thirlwall, 1998; Thirlwall and McCombie, 1997). According to this analysis, excess capacity and unemployment are deeply entrenched in the dynamics of capitalism and exogenous demand embodied in exports holds a key position in capital accumulation and growth.

Nevertheless, as a recent contribution by Moudud (2000) has shown, the post-Keynesian Thirlwall approach does not capture the richness of Harrod’s analysis of growth and cycles. In building his export-led model, Thirlwall started from the Harrod static analysis of the open economy multiplier (1933) and tried to make it dynamic, even if with questionable results. The dynamic aspects of Thirlwall’s model could be found mostly in the effort of formalizing
Kaldor’s regional growth rather than Harrod’s dynamics. Additionally, his successive balance of payments constrained growth model followed basically a static approach because the analysis was confined to levels rather than paths of growth. As Moudud (2000, p. 1) observes: “…Harrod’s perspective differs from Thirlwall’s. This distinction reflects the one between Keynes’ static perspective and the dynamic tradition of the Physiocrats, Marx, von Neuman and Harrod”.

This paper develops an open economy macrodynamic model on Harrodian lines. The analysis has a twofold aim: i) exploring how the open economy growth framework changes when the typical Harrodian instable dynamics explicitly enters the analysis and comparing results with Thirwall’s approach; ii) in a reverse way, investigating whether and how the Harrodian instable dynamics changes when net exports enter the analysis; this point follows from Harrod himself (1973) who suggested that the expansionary policy embodied in a net export increase could help to reduce the gap between the warranted growth path and the actual one. Our model is built on a recent formal version (Sportelli, 2000) of Harrod’s works on dynamics where the economic growth and business cycle are jointly explained. The model is able to yield at least three important results: i) technical progress has a crucial role in defining the possibility of export-led growth dynamics; ii) discrepancies between the actual, expected and technical progress rate of growth are responsible for cycles; iii) instability of cycles is smoothed in open economy.

The paper is organised as follows. Section 1.1 briefly outlines the demand-orientated approach to growth in open economy according to a line of thought starting from Harrod’s trade multiplier and moving to the Kaldor export-led growth and the Thirlwall balance of payments constrained growth models. Section 1.2 focuses on the link between economic dynamics and foreign trade in Harrod, trying to offer a different point of view with respect to Thirlwall’s interpretation. Section 2 is devoted to set out the assumptions of the model and its basic equations. Section 3.1 describes Harrod’s basic model which is, in our view, an under-determined dynamical system. Therefore, one possible way to complete the model is suggested in section 3.2. Section 3.3 deals with the mathematical analysis of a specified four dimensional dynamical system, while sections 3.4 and 3.5 provide the qualitative study of its fixed points and the numerical simulations of the results. Finally, section 4 contains some concluding remarks.

1.1 From Harrod’s trade multiplier to the Kaldor-Thirlwall export-led growth

In chapter six of his *International Economics* (1933), Harrod examined forces underlying the balance of payments (BP) equilibrium. Harrod’s starting point was the critique of the classical theory of BP adjustment, i.e. the Gold Standard approach. According to Harrod, the price-species flow mechanism postulated by the Gold Standard doctrine was unrealistic and inadequate for equilibrating BP. In fact, when different typologies of goods were introduced
into the analysis (tradable, semi tradable and non tradable), the classical mechanism based on price movements was seriously compromised and changes in the level of economic activity became a more plausible explanation of BP adjustment. Harrod proposed an alternative BP adjustment mechanism based on the foreign trade multiplier. Under the hypothesis of production ($Y$) consisting in goods domestically consumed ($C$) or exported ($E$) and under the assumption of income entirely spent in consumption goods produced domestically or imported ($M$), the equilibrium between expenditure and income implied the balance between exports and imports:

$$Y = C + E = C + M \Rightarrow E = M.$$  

As $M = mY$, where $m (= \text{constant})$ is the marginal propensity to import, it followed that $Y = E/m$. Since $0 < m < 1$, for given values of $E$ and $m$, the income level was determined via the coefficient multiplier $1/m$.

The Harrodian foreign trade multiplier mechanism of BP adjustment well captured the international transmission of economic cycles. If the classical theory depicted the world economy as a static context in which BP surplus and deficit were reallocated between countries via equilibrating gold flows and price movements, Harrod’s framework could explain the simultaneous involvement of all countries in the same deflationary or expansionary phase of the international cycle through generalized changes in the volume of exports. The theory of BP adjustment illustrated by Harrod was the first explanation of external constraint on Keynesian lines.

Kaldor’s successive effort to justify the international differences in growth rates has been interpreted as the construction *de facto* of an export-led growth framework which is the translation of Harrod’s foreign trade multiplier theory into a dynamic form. The Kaldorian analysis of growth in open economy has not been formalised through a mathematical model by the author. Actually, the basic ingredients of his informal model can be found in some works published from the second half of the 1960s onwards. In the *Cause of the Slow Rate of Economic Growth in the United Kingdom* (1966), Kaldor remarked the positive correlation - empirically observed for the majority of industrialised countries - between the growth rate of the manufacturing industry and the growth rate of the whole economy. In *The Case for Regional Policies* (1970), Kaldor explained that positive relationship in a coherent analytical framework: manufacturing activities were characterized by increasing returns and this circumstance justified the positive link between productivity growth and output growth, the so called “Verdoorn law” (1949). According to Kaldor, this association had to be interpreted in dynamic terms as “circular and cumulative causation” à la Myrdal (1957): if two regions (or two countries) open to international trade, the region with a more developed manufacturing industry will capture cumulative advantages deriving from the expansion of the market for its products and from the resulting reduction of its production costs, inhibiting the development of the same manufacturing activities in the less advanced region. If the classical theory of international trade emphasized mutual gains from trade under the hypothesis that constant returns to scale led to a convergence of growth rates, the assumption of increasing returns and the general principle
of cumulative causation changed the perspective of adjustment by introducing the possibility of increasing divergence in comparative costs and in growth rates.

Thus the interaction between increasing returns and cumulative causation allowed Kaldor to offer an explanation of the positive link between the growth rates of manufacturing industry and growth rates of the whole economy. The further consideration that in open economy exports represented the most important component of autonomous demand for industrial product, conducted de facto to an export-led growth model 1.

Dixon and Thirlwall (1975) proposed a formal modelling of Kaldor’s ideas. The basic equation of their model established that the growth rate of the economy was governed by the growth rate of exports. Exports were set up as a multiplicative function of the level in domestic prices, foreign prices and foreign income, so that the growth rates of the variables entered the function in additive terms. A domestic price equation fixed the negative link between domestic prices and productivity, and a productivity equation set up a positive relationship between productivity and output (Verdoorn law). In this way, the model was circular and cumulative: an increase in exports pushed up output, productivity, price competitiveness, exports, etc.

The Dixon-Thirlwall model assumed a regional context with no balance of payments equilibrium condition. Successively, Thirlwall (1978) introduced the balance of payments constraint explicitly. In this last case, the idea that exports could determine a virtuous circle with growth still remained, but this possibility was conditioned by the role of imports in the long run equilibrium condition of balance of payments. In other words, if a country incurred in a deficit of balance of payments before reaching the full utilization of productive capacity, this circumstance would need a contraction of demand with detrimental effects on investment, productivity, competitiveness and, therefore, a vicious circle would emerge. On the contrary, an increase in demand beyond the level consistent with the full utilization of productive capacity without incurring in difficulties of the balance of payments would imply the activation of a virtuous circle: capital accumulation, rising productivity, stronger competitiveness, etc.

In formal terms, the starting point of Thirlwall’s balance of payments constrained growth model was the equilibrium in current account:

\[ P_d E = P_f M F \]

where \( E \) and \( M \) are the volume of exports and imports respectively, \( P_d \) is the price of exports in domestic currency, \( P_f \) is the price of imports in foreign currency and \( F \) is the exchange rate. In terms of growth rates (indicated by small letters), the former becomes:

\[ \dot{P} E = \dot{P} F \]

1 Actually, once we allow for the activation of Hicks’ super-multiplier (1950), exports represent the only real autonomous component of demand (Kaldor, 1970).
After expressing exports and imports as multiplicative functions of the prices of exports and imports, and foreign and domestic incomes, respectively, Thirwall arrived to define the balance of payments equilibrium growth rate:

\[ p_d + e = p_f + m + f \]

where:
- \( p_d \) = price elasticity of demand for exports (\( h < 0 \));
- \( f \) = cross elasticity of demand for imports (\( f > 0 \));
- \( \delta \) = cross elasticity of demand for exports (\( \delta > 0 \));
- \( \psi \) = price elasticity of demand for imports (\( \psi > 0 \));
- \( \epsilon \) = income elasticity of demand for exports (\( \epsilon > 0 \));
- \( \pi \) = income elasticity of demand for imports (\( \pi > 0 \));
- \( z \) = growth rate of foreign income.

If we assume that relative international prices are stable in the long run\(^2\), we arrive to Thirlwall’s most important result:

\[
y_b = \frac{p_d(1+\eta - \phi) - p_f(1-\delta + \psi) - f(1+\eta + \psi) + \epsilon(z)}{\pi}
\]

According to Thirwall, this last expression represents the simple Harrodian foreign trade multiplier expressed in a dynamic form. It would explain the growth experiences of many countries: differences in income elasticity of demand for imports and exports across countries would signal different structural abilities in relaxing the external constraint to growth and, therefore, would explain international differences in growth rates.

### 1.2 Harrod’s dynamics and foreign trade

Is the simple Thirlwall’s formula exhaustive with regard to the role played by foreign trade in the Harrod dynamics? As McCombie (1998) has remarked, Harrod never integrated formally his two most important contributions to the development of Keynesian theory, that is: the extension to open economy of

\[^2\text{In an argumentative reply to Cairncross, Kaldor (1978) remarked that prices’ elasticity was not relevant in explaining the poor growth performance of UK. On the contrary, the high income elasticity of demand of UK for foreign goods signalled a real constraint to growth. So, according to Kaldor, if the explicative power of income elasticity is higher than the price-elasticity prediction, it means that we are in a Keynesian-Harrodian world and not in Ricardian world. Thirlwall agrees with this.}\]
the multiplier concept, and the dynamic interpretation of the General Theory. As illustrated in the section 1.1, starting from the Harrodian static analysis of foreign trade multiplier, Thirlwall derived the balance of payments equilibrium growth rate. According to McCombie (1998), the Thirlwall procedure represents the first attempt at reconciling the two important pieces of analysis which Harrod never incorporated formally. As already remarked by Moudud (2000), we believe that Thirlwall’s contribution essentially follows a static approach and does not account for important implications of Harrod’s dynamic investigation. Thirlwall’s analysis, for example, does not consider the relationship between the level of capacity utilization and investment which in Harrod is crucial in determining the link between warranted and actual rates of growth. If export growth induces a pressure on productive capacity above its normal level, how does the Thirwall’s model respond? Following an instable growth path according to Harrod? Or is the underlying hypothesis of Thirlwall’s model the maintaining of excess capacity? And in this case, why the investment would not decline? It seems to us that some crucial dynamic interactions existing in Harrod’s analysis are undervalued in Thirlwall’s approach.

For this reason, here we choose a different perspective (in some way, the reverse procedure followed by Thirlwall). We start from Harrod’s dynamics in a closed economy and follow some clues of the link between dynamics and foreign trade Harrod himself revealed in some of his works. For example, in Towards a Dynamic Economics (1948), Harrod suggested that the “fundamental equation” indicating the warranted growth rate has to be emendated when foreign trade is considered:

\[ G_w C_r = s - b \]

where: \( G_w \) = the warranted growth rate; \( C_r \) = the desired capital coefficient; \( s \) = share of income saved; \( b \) = the balance of trade expressed as a fraction of income.

According to Harrod, in a closed economy the divergence between the warranted rate of growth (the equilibrium growth path consistent with desired saving rate and desired capital coefficient) and the actual one (\( G \)) generates instability. In fact, if \( G_w < G \), desired savings are lower than investments and this situation creates inflationary pressures and/or a reduction in inventories that push up investments determining a further divergence between \( G_w \) and \( G \). On the contrary, if \( G_w > G \), desired savings prevail over investments and this circumstance activates depressive forces which push the system further below the equilibrium growth path. According to Harrod, foreign trade could reduce the divergence between \( G_w \) and \( G \) and therefore could moderate cyclical instability. For example, in the case \( G_w > G \), a surplus of current account push down \( G_w \) and this reduces the gap between \( G_w \) and \( G \). In this recessionary situation, a trade balance improvement will drive \( G \) towards \( G_w \); short run unemployment will decline, although long run unemployment will
increase. This example shows that a surplus of current account could be detrimental for long run growth. This result is inconsistent with Thirlwall’s model but is perfectly coherent with Harrod’s dynamics extended to an open economy.

Here we propose a growth model embodying Harrod’s instability principle to investigate the impact of foreign trade properly.

2. Preliminary assumptions and basic equations

2.1 List of symbols

Here we assume an open *laissez faire* economy. We make use of the following notations:

- $I_j$, ex ante investment including both equipment and desired inventory stocks;
- $I$, ex post investment including both equipment and effective inventory stocks;
- $S$, ex post saving;
- $E$, exports;
- $M$, imports;
- $X = E - M \geq 0$, balance of trade;
- $Y$, effective demand;
- $S/Y = \Sigma$, share of income saved;
- $x = X/Y$, the ratio of balance of trade to income (or simply, the net export rate);
- $G = \dot{Y}/Y$, the (actual) rate of growth of domestic income;
- $Y^e$, expected demand;
- $G_w = \dot{Y}^e/Y$, the (warranted) expected rate of growth of aggregate demand;
- $G_n$, technical progress (a rate of growth);
- $G_f$, the rate of growth of the foreign demand.

All the variables are defined in continuous time and considered in real term. So from now on, a dot on the variable will indicate the derivative operator $d/dt$.

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3 Some of our symbols are unconventional. This because most models usually assumes as constant what here we set as variable.
4 Likewise to Hahn (1990, p. 23) we interpret Harrod’s warranted rate of growth as a variable rate defined on the basis of firm’s (rational) expectations. A similar interpretation has been suggested by King and Robson (1992, p. 44) and already adopted in Sportelli (2000, p. 169).
2.2 Harrod’s instability principle

As we consider an open *laissez faire* economy, ex post investment equals ex post saving minus the balance of trade, i.e.

\[ I = S - X \]  (1)

In accordance with Harrod’s definition of capital coefficient, which includes both circulating and fixed capital, we set:

i) \( C_r = \frac{I_t}{\hat{Y}^e} \) as the desired capital coefficient, i.e. “the requirement for new capital divided by the increment of output to sustain which the new capital is required” (Harrod, 1948, p. 82);

ii) \( C = \frac{I}{\hat{Y}} \) as the actual capital coefficient, i.e. “the increase in the volume of goods of all kinds outstanding at the end over that outstanding at the beginning of the period divided by the increment of production in the same period” (Harrod, 1948, p. 78).

**Assumption 1:** In every period \( t \), given the current desired capital coefficient, firms’ investment decisions are taken looking at the expected change in demand \( \hat{Y}^e \). It follows that ex ante (justified) investment will be

\[ I_j = C_r \hat{Y}^e \]  (2)

Even if justified by a “rational” expected \( \hat{Y}^e \), ex ante investment may differ from the realised one when, ex post, inventories are taken into account. Inventory changes will happen whenever the actual change of demand \( \hat{Y} \) differs from the expected change \( \hat{Y}^e \). This allows us to state the following:

**Proposition 1:** If, at a given time \( t \), \( \hat{Y} \gtrless \hat{Y}^e \) (\( \hat{Y} \) and \( \hat{Y}^e \) both positive), the actual (ex post) investment \( I \) will be smaller/greater than ex ante \( I_j \) and consequently the actual desired capital coefficient \( C_r \) will result greater/smaller than the actual \( C \).

**Proof:** As we consider investment including both equipment and inventory stocks, the inequality \( \hat{Y} \gtrless \hat{Y}^e \) implies a storehouse gap. Specifically, if \( \hat{Y} > \hat{Y}^e \), firms’ stocks will be, on average, below the desired level and \( I < I_j \);
conversely, if $\dot{Y} < \dot{Y}^c$, it will be $I > I_j$. Therefore Harrod's definition of $C_r$ and $C$ yields:

$$\left( \dot{Y} \equiv \dot{Y}^c \Rightarrow I \equiv I_j \right) \Leftrightarrow C_r = \frac{I_j}{Y^c} \equiv C = \frac{I}{\dot{Y}}$$

Let us suppose $\dot{Y} > \dot{Y}^c$. In this case, each firm rationally will decide new investments to restore stock levels and to increase, if necessary and profitable, its productive capacity to make it consistent with new levels of production. This implies an increase in $I$ (at least in its inventory component), which will create, by means of the monotonic multiplier effect, a new and larger value of $\dot{Y}$. Since $I$ and $\dot{Y}$ are both changing surely not proportionally, $C$ too tends to change. Furthermore, as firms are careful to acquire any new information generated by the system to forecast future demand, the level of demand currently expected (the new $Y^c$) will differ from its past value. This is because agents' rational behaviour requires that an erroneous expectation must be changed (Fazzari, 1985). The comparison between the new $Y^c$ with the perceived current level of demand enables firms to define the actual expected change in demand $\dot{Y}^c$. In a period of rising business activity, the new $\dot{Y}^c$ will be positive and presumably greater than its previous value. Under these conditions a new and higher amount of investment will be decided according to Eq. (2).

If we assume a time interval where $\dot{Y}^c$ is increasing (because at each $t$ $\dot{Y} > \dot{Y}^c$), the justified investments will grow inasmuch as $C_r$ too is increasing. In fact, the new higher investment decisions are justified by the need for filling existing storehouse gaps, and by the need for restoring stocks and new equipment to the level which is needed to sustain the new value of $\dot{Y}^c$. Whenever the expected demand increases are weaker than those in actual aggregate demand over the given time path, $I_j$ will undergo acceleration. Therefore, as investment involving fixed capital cannot be carried out immediately, the $I_j$ acceleration leads to a rising distance between $I$ and $I_j$. But if $I_j$ is pushed forward $I$, our Proposition 1 implies that $C_r$ too will be pushed ahead of $C$. So that, while $C$ follows $C_r$ because of the time required building new productive capital, changes undergone by $C_r$ are path dependent on the gap $\dot{Y} - \dot{Y}^c$.

Let us note that, when $\dot{Y} < \dot{Y}^c$ is assumed, our conclusions are similar although our reasoning needs to be reversed. In other words, what is different is the difference between $C_r$ and $C$: a rising business activity generates a

---

5 In other words justified investments will evolve according to the following derivative: $\dot{I}_j = C_r \dot{Y}^c + C \dot{Y}^c$, where the terms on the right-hand side have the meaning explained above. Details on this point can be found in Sportelli (2000, pp. 173-173).
sequence of \( C_r - C > 0 \), and a falling business activity generates a sequence of \( C_r - C < 0 \).

The \( C_r \) path dependence (in either direction) on the gap \( \dot{Y} - \dot{Y}^e \) can be interpreted as the first intrinsic component of Harrod’s instability principle. This allows us to set the following:

**Assumption 2:** The current mean level of the desired capital coefficient is a function of the current discrepancy between the effective and the expected change of demand.

Formally, considered the gap \( \dot{Y} - \dot{Y}^e \) as a proportion of the actual level of demand (i.e. \( \frac{\dot{Y} - \dot{Y}^e}{Y} \)), we can write:

\[
C_r = \Phi \left( \frac{\dot{Y} - \dot{Y}^e}{Y} \right) = \Phi \left( G - G_w \right)
\]

such that \( \Phi' > 0 \) and \( \Phi(0) = C^* > 1 \) because \( G = G_w \) (or equivalently \( \dot{Y} = \dot{Y}^e \)) implies \( l_f = I \) and, consequently, \( C_r \) equal to its equilibrium value \( C^* \) whenever the actual demand change is exactly the one firms will check, on average, ex post.

There is a second important component of Harrod’s instability principle which arises from firms’ rational reaction to a disequilibrium condition. As we said earlier \( I \leq l_f \) implies an increasing (decreasing) \( I \) which will give rise to a new \( \dot{Y} \geq 0 \) by means of the multiplier effect. Therefore, if we assume a time interval such that \( I_f - I \geq 0 \) always holds, then a sequence of positive (negative) \( \dot{Y} \) will be generated and, if the distance between \( I_f \) and \( I \) increases (in absolute value), then \( \dot{Y} \) dynamics will be accelerated (slowed down). This occurs because changes in the output to bring inventories to their desired level together with higher (lower equipment orders, force all firms to accelerate (to slow down) their production rates. Consequently the economy will experiment a period of rising (falling) rate of growth of income. This conclusion leads us to our third assumption:

**Assumption 3:** Changes in the growth rate of income are dependent on the gap between ex ante (justified) and ex post (actual) investments.
In accordance with Alexander’s postulate\(^6\) (1950, p. 263), if we define as \(U\) the difference between \(I_j\) and \(I\), given Equations (1) and (2), we can set:

\[
U = I_j - I = C_r \hat{Y}^r - (S - X)
\]

i.e. (by measuring the gap \(I_j - I\) relatively to the current level of output),

\[
u = \frac{U}{Y} = C_r G_w - (\Sigma - x)
\]  \hspace{1cm} (4)

Given Eq. (4) our Assumption 3 can be formally stated as

\[
\dot{G} = F(u) = F \left( C_r G_w - \Sigma + x \right)
\]

which becomes, taking into account Eq. (3),

\[
\dot{G} = F \left( \Phi(G - G_w)G_w - \Sigma + x \right)
\]  \hspace{1cm} (5)

As ex post investments exhibit some stickiness along their dynamic path, inasmuch as some part of investments includes changes in productive capacity, we shall assume \(0 < F'(u) < 1\). Furthermore, the function \(F\) will be such that \(F(u) \geq 0\) if \(u \geq 0\). When the equality holds, it is important to point out that the expression \(\Phi(G - G_w)G_w - \Sigma + x = 0\) implies \(I_j = I\) and therefore, \(\hat{Y} = \hat{Y}^r \iff G = G_w\). Since \(\Phi(0) = C^r\), it follows that \(C^r G_w = \Sigma - x \iff C^r G = \Sigma - x\). This result agrees with Harrod’s fundamental equation concerning the steady state growth. But, unlike Harrod (1948, p. 105), our steady growth solution has nothing to do with the dynamic equilibrium of the economy\(^7\). It is only able to define an equilibrium relationship between \(G\) (or \(G_w\)) and ex post investments, which are variable in our model as we shall see very soon. Indeed, this result suggests the possibility of transient multiple equilibria between \(I_j\) and \(I\) which agree with a steady state growth\(^8\).

---

\(^6\) This postulate was originally formalised in discrete time for a closed economy. Firstly it has been restated in continuous time by Sportelli (2000, p. 174).

\(^7\) Differences between ours and Harrod’s conclusions arise from the different definition of \(G_w\). We have to point out that, if we should abide by Harrod’s (1948) definition of \(G_w = (\Sigma - x)C_r\), our Eq. (5) would be always equal to zero. See Besomi (1998, pp. 51-53) for details on questions concerning Harrod’s \(G_w\).

\(^8\) An analogous result is already in Sportelli (2000).
2.3 The saving rate dynamics

There is a wide agreement among Economists about the existence of “a virtuous cycle in which high growth promotes high saving, and high saving in turn promotes fast growth” (Stiglitz, 1999, p. ix). Nevertheless, the causality relationship between aggregate saving and macroeconomic growth remains ambiguous on theoretical as well as on empirical grounds (Deaton, 1999, p. 38).

To examine this question, we begin by recalling that savings are, in the short run, the residual between earnings and consumptions. The consequence is that some part of the aggregate saving at any given time may be either undesired or unexpected or both. Furthermore, since we assume $S/Y = \Sigma$ as a variable, unlike most of macrodynamic models (both Keynesian and neoclassical) postulate, we have to distinguish explicitly between the level of saving and the saving/income ratio. This distinction has a crucial importance in understanding the role of savings into a dynamical framework because changes in the level of saving prevalently depend on changes that income may undergo over time, while changes in the saving/income ratio only depend on the speed at which $S$ may vary when the income level is changing.

We know that an economy governed by the effective demand principle is sensitive to the level of aggregate saving. Consequently, all the components of those savings are important. Canonical rules of agents’ rational behaviour only suggest that people are able to plan the desired component of saving. This implies that whenever income differs from its expected level, the rate of change of saving will be higher than the rate of change of income. The reason is simple. When the income level is above its expected level, $S$ increases quicker than $Y$. In the first place this happens because household consumption are stable as they are the result of experienced welfare levels linked with past, current and sometimes future expected earnings. Therefore, unexpected income increases cannot change immediately consumption expenditure. Secondly, when firms experience large unexpected increases of profit margins, they are inclined to devote their higher funds to internally financed investments, or to dividends distributed to shareholders. Whatever their choice may be, the result will be a quicker rise of $S$. Conversely, if income decreases below its expected level, the households’ saving decreases quicker than income because a saving drop is preferred to an immediate cut in consumption. On the firms’ side, unexpected reductions of profits margins normally lead firms to cut the dividend distributed to shareholders, before revising their investments plans. The effect is an unexpected reduction of households’ earnings and so a lower households saving.

Looking at the economic system as a whole, what we described earlier leads to postulate that the elasticity of aggregate savings with respect to income is typically greater than one, rather than equal to one as most economic model implicitly suppose by taking $S/Y$ as a constant.

---

9 We have to note that the share of undesired or unexpected saving is negative in this case.
In the real world the elasticity of saving shows the tendency: i) to rise above one whenever profits or household earnings increase over their expected level (because $\frac{\dot{S}}{S} > \frac{\dot{Y}}{Y}$); ii) to be close to one if profits or household earnings are close to their expected level (because $\frac{\dot{S}}{S} \cong \frac{\dot{Y}}{Y}$); iii) to rise again, moving away from one, when household earnings or profits fall below their expected level (because $\frac{\dot{S}}{S} < -\frac{\dot{Y}}{Y}$) (Piscitelli and Sportelli, 2004, p. 14).

Our conclusions seem to be consistent with Harrod’s intuitions (1939, p. 21) because our $\Sigma$ varies independently of the changes $G$ may undergo in the different phases of the cycle$^{10}$.

As shown by Sportelli (2000, pp. 176-179), unexpected changes of income come from the dynamics of Harrod’s natural rate of growth$^{11}$. Here unlike Harrod, our $G_n$ neglects the contribution resulting from population growth and is interpreted, more generally and according to its modern meaning, as the growth rate of technological progress. Since technical progress enhances the labour productivity which improves in its turn profit margins and household earnings, we assume the following:

**Assumption 4:** In the economic system the saving rate varies over time. Its variations derive from unforeseen differences between the technical progress dynamics and the expected rate of growth of income.

Assumption 4 can be formally stated as

$$\dot{\Sigma} = \Psi \left((G_n - G_w, \dot{G}_w) \right)$$

(6)

where $\partial \Psi / \partial (G_n - G_w) > 0$ and $\partial \Psi / \partial \dot{G}_w \cong 0$ iff $\dot{G}_w \cong 0$.

The variable $\dot{G}_w$ accounts for the effect on $\Sigma$ of adjusting expectations once discrepancies between the forecast and current change of income have been observed. The presence of $\dot{G}_n$ among the arguments of the function $\Psi$ may be interpreted as a hysteresis variable affecting the saving rate in the short run. This is because either positive or negative shocks on aggregate savings due to sudden changes in income cannot be immediately reversed in the subsequent time period. Some memory of past shocks is likely to remain

---

$^{10}$ On p. 25 of his 1939 paper Harrod wrote that the saving/income ratio is inclined “to vary with a change in the size of income, but a change in the rate of growth at a given point of time has no effect on its size”.

$^{11}$ “The rate of advance which the increase of population and technological improvements allow” (Harrod, 1948, p. 87).
over time. But, in the long run, these effects tend to average out and compensate each other (Piscitelli and Sportelli, 2004, p. 15).

2.4 The balance of trade dynamics

In accordance with Harrod, our dynamic equation explaining the changes that the actual rate of growth may undergo over time embodies the trade balance as a proportion of income. This is of no surprise unlike what McCombie (1998, p. 215) and Moudud (2000, p. 5) seem to believe about Harrod’s model when it is extended to an open economy. Here instead the share of the trade balance into the “fundamental equation” arises spontaneously from the formal version of the instability principle suggested by Alexander (1950)\(^{12}\).

What is really important to point out here is that it is the share of trade balance which affects the growth rate and not the export dynamics alone. This implies that we have to focus our attention on the elements affecting both import and export dynamics.

As we well know, there are two crucial variables which determine the behaviour of exports: the demand of foreign economy as a whole and the relative prices. Only if we consider a static context exports are the most important, since their increase warrants the trade balance improvement and the increase of domestic income by means of the foreign trade multiplier, though, at the same time, imports rise because the domestic income is rising.

In a dynamic framework what has a crucial importance is the speed at which the foreign demand changes with respect to the domestic income and the relative prices. If we consider the rate of growth of the foreign demand \( \dot{G}_f \) relatively to the domestic growth rate of income, net exports will increase iff \( \dot{G}_f > G \) (Harrod, 1948, p. 108)\(^{13}\). But, Harrod observes, if “other things” remain equal. What are these other things it is not difficult to understand. Although Harrod’s reasoning may seem sometime farraginous, we can infer that “other things” have to do simply with the relative price dynamics.

When at a given time \( \dot{G}_f > G \), the speed at which exports may rise depends on the sensitivity of export with respect to \( G \). As the domestic income increases, imports rise too and the quickness of their changes depends on the sensitivity of imports with respect to the internal growth rate. If we break off at this point our reasoning, we should assert that the ratio of the balance of trade to income \( x = (E - M) / Y \) (i.e. the net export rate) might rise over time if the speed at which \( E \) rises is so high, after deducting the speed at which \( M \) is rising, to remain higher than the speed at which \( Y \) is

\(^{12}\) We recall the reader that Alexander’s dynamic equation of \( \dot{G} \) received, though some criticism, an explicit approval by Harrod (1951, p. 263).

\(^{13}\) Implicitly Harrod assumed that there was no difference in the income elasticity of demand for exports and imports across countries.
rising too. In other words, looking at the derivative of \( x \) with respect to time, we can infer that \( \dot{x} > 0 \) if \( [\dot{E} - \dot{M} - G(E - M)]/Y > 0 \). Certainly this inequality may hold, but with the same certainty we can state that it cannot persist over time. The reason is that when \( Y \) rises, sooner or later, domestic prices increase not only because the system is getting near the full employment threshold, but also because there may be structural knots in the system such that prices rise before the full employment is reached. It is obvious that domestic prices dynamics must be seen comparatively to the changes foreign prices may undergo over time, nevertheless if the growth of domestic technical progress is high enough to slow down the increase of internal prices with respect to the foreign ones, then changes in the net export share may preserve a positive sign for a relatively long time. In fact, by improving labour productivity, the technical progress reduces the unit cost of domestic tradable goods and their competitiveness may persist on the foreign markets\(^{14}\).

After all that, we are able to state that three are the elements affecting the net export rate in a dynamic framework: the foreign demand, the domestic income and the technical progress rates of growth. This justifies the following:

**Assumption 5:** Relative changes of the ratio of the trade balance to (domestic) income depend on the dynamic interaction of \( G_f \), \( G_n \) and \( G \).

Assumption 5 can be formally stated as:

\[
\frac{\dot{x}}{x} = \Xi(G_f, G_n, G) \tag{7}
\]

where the function \( \Xi \) is such that \( \partial \Xi/\partial G_f > 0 \), \( \partial \Xi/\partial G_n > 0 \), \( \partial \Xi/\partial G < 0 \). Furthermore we shall assume \( \Xi(G_f, 0, 0) < 0 \) because a constant domestic production without technical progress cannot face a foreign demand without undergoing increasing domestic prices. So that, the range of domestic goods that are competitive abroad reduces and, correspondingly, an increasing range of foreign goods gains competitiveness in the home market.

\(^{14}\) It is implicit here that we neglect any other monetary factor which may affect prices dynamics.
3. An open economy macrodynamic model on Harrodian lines

3.1 Harrod’s basic model

If we put together Equations (5), (6) and (7) the result is a basic model on Harrodian line which is more complex than the ones we can find amongst the wide number of models inspired to Harrod’s dynamic theory. Furthermore, as our basic model is an extension of the one suggested by Sportelli (2000) for a closed economy, it is the same an under-determined dynamical system:

\[
\begin{align*}
\dot{G} &= F\left( \Phi(G - G_w)G_w - \Sigma + x \right) \\
\dot{\Sigma} &= \Psi \left( (G_n - G_w), \dot{G}_w \right) \\
\dot{x} &= \Xi \left( G_f, G_n, G \right) x
\end{align*}
\]

because it includes three equations and seven variables \((G, G_w, \Sigma, x, \dot{G}_w, G_n, G_f)\).

Although same variables such as \(G_n\) and \(G_f\) might be considered as exogenous and constant, surely \(G_w\) (and \(\dot{G}_w\)) cannot be if we want to preserve same consistency with our basic model and Harrod’s thought on dynamics. In fact, Harrod himself wrote in his 1939 (p. 28) paper: “As actual growth departs upwards or downwards from the warranted level, the warranted rate itself moves, and may chase the actual rate in either direction”.

This clearly means that, in Harrod’s view, \(G_w\) (and so \(\dot{G}_w\) too) is a variable.

Therefore, we can confirm that the system (H) remains (like the one formalised for a closed economy) an under-determined system. As a consequence, we disagree completely with McCombie (1998, p. 215) who states that “Harrod’s model is over-determined since there is nothing to ensure that the warranted and natural growth rates would be equal except by coincidence”. This opinion is founded on formal assumptions misinterpreting Harrod’s instability principle, since both \(C\) (or \(C_i\)) and \(\Sigma\) are considered as exogenous and constant.

The only one variable here we shall assume as exogenous and constant is the rate of growth of foreign demand. This is a usual assumption we share with all one country dynamic models of international trade.
3.2 \ The completion of the model

Our formalization of Harrod’s macrodynamics embodies two variables $G_w$ and $G_n$ which need to be endogenous to make the basic model able to describe the dynamic behaviour of an economic system. If either $G_w$ or $G_n$ were exogenous, not only Harrod’s instability principle would simply coincide with the mathematical notion of dynamic instability, but the model should become an algebraic tool able to predict changes in $G$ coming from given discrepancies between justified (ex ante) and effective (ex post) investment rates (i.e. from the gap $(I/Y) - (S - X)/Y$).

Therefore, as far as the $G_w$ dynamics is concerned, since some element of bounded rationality is implicitly contained in the agents’ behaviour here we assumed, we need an hypothesis able to describe the evolution of expectations which may be consistent with the fact that agents do not take their decisions at same time. That is the reason why the simplest hypothesis we can make is the following:

**Assumption 6:** The expected rate of change of the aggregate demand is a continuous function of the weighted average of all past rates of change of effective demand.

That is,

$$G_w = \int_{-\infty}^{t} \frac{1}{T} e^{-(t-\tau)/T} G d\tau$$ \hspace{1cm} (8)

Where the mean time lag $T$ assigns more or less weight to all past values of $G$. As firms are able to infer the changes in the actual level of demand by periodic checks of their stocks (at least once in a period), we shall assume $T \leq 1$.

As far as the $G_n$ dynamics is concerned, we have to recall that our $G_n$ is meant as a technological progress rate of growth.

It is well known that the evolution of technology is driven by the technical knowledge which is itself a kind of capital good exploited in combination with other factors (labour and capital equipment) to produce final output. Furthermore, knowledge can be stored over time and can be accumulated through investments in R & D and any other knowledge-creation activity. This process involves the sacrifice of current resources in exchange for future benefits of productive efficiency and product innovations.

The efficiency of production process may also be induced from the experience of producing new capital goods. This last phenomenon is nothing
other than the well known Arrow’s (1962) “learning by doing”. Learning by
doing is external to the firms and is related to the general concept of
 technological knowledge which is assumed to be a public good. Despite the
fact that this assumption has caused some criticism (e.g. Romer, 1986, 1990)
since knowledge is often lacking in complete non-rivalry and non-excludability
characters, empirical studies have shown that investments in new capital
goods create a spillover effect (Cohen and Levinthal, 1989; Offerman and
Sonneman, 1998). Therefore, it often happens that new ideas embodied in
investment projects realised by one firm (or industry) generate externalities
for other firms (or industries), which learn from these ideas and adapt them
for their own business. This other phenomenon tightly linked with the learning
by doing is dubbed “learning by watching” (King and Robson, 1992, 1993).
But we know that technical competences and ability to experiment and adopt
new methods of production depend on the resources individuals devote to
investments in education, i.e. in human capital (Greiner and Semmler, 2002).

Looking at the economic system as a whole, all we said earlier shows that
the evolution the technological progress may undergo over time crucially
depends on the resources invested in physical capital goods, technical
knowledge (which is a disembodied capital good) and education. These
ingredients enhance technology and favour labour productivity increases. All
that leads us to think that the variable which affects most the technical
progress dynamics is the aggregate investment rate in the economic system.

Kaldor (1957) was the first who postulated the existence of a technical
progress function disjoined from an aggregate production function. This idea
has already been introduced by King and Robson (1992, 1993) into a one
sector neoclassical growth model and adapted by Sportelli (2000) to a
Harrodian model of cyclical growth. Here we restate this last approach by
assuming that technological progress depends on the resources the system
devotes to investments of all kind, i.e. new capital goods, knowledge and
human capital. Since ex post the aggregate saving rate embodies all kind of
investment, we state the following:

**Assumption 7:** In our economic system, dynamics of technological
progress is a continuous non-monotonic function of the share of income
saved and devoted to investments of all kind.

Formally we shall write:

\[ G_n = G_n (\Sigma) \] (9)

Looking at our theoretical consideration, we think of the function \( G_n \) as being
increasing and non-linear. Nevertheless, since the unity is the least upper
bound of the saving rate, it is implausible to suppose the function \( G_n \) may be
still raising when the saving rate is close to one, i.e. when consumptions are
ceasing to exist in the system. Therefore, likewise Kaldor’s technical progress
function, we shall provide our $G_n$ with the following analytical property: $\exists \Sigma < 1$
(and far enough from one) such that $G_n(\Sigma)$ is a local maximum. This implies
that $G_n$ may be rising only if $\Sigma < \bar{\Sigma}$. But when $\Sigma > \bar{\Sigma}$ the rate of growth of the
technical progress begins to decrease not only because decreasing returns
may characterise the learning by watching\textsuperscript{15} as well as any other research
activity, but also because the complementary reduction of consumption rate
(given the net export rate as a constant) might be a force making for a
slackening productivity dynamics. So that the $\Sigma$ value may be interpreted as
an optimum saving rate from the social-welfare standpoint or, equivalently, as
the maximum share of resources devoted to knowledge-creation activities
which are socially and efficiently sustainable.

3.3 The specific dynamical system

Up to now we have considered generic function to build what we defined
the Harrod basic model. To simplify our qualitative as well as numerical
analysis, we have to specify those functions preserving all their qualitative
properties. There might be different ways to specify the model. Here we
chose the simplest where the functions are linear if possible. Therefore we
propose the following functional forms:

- Changes in the rate of growth of income

$$F(C, G_w - \Sigma + x) = \alpha (C, G_w - \Sigma + x)$$

where $0 < \alpha < 1$ because of the stickiness affecting the investment
components which involve changes in the productive capacity.

- Changes in the saving rate

$$\Psi((G_n - G_w), \dot{G}_w) = \varepsilon (G_n - G_w) + \delta \dot{G}_w$$

where $\varepsilon, \delta > 0$ are sensitivity parameters. As the gap $(G_n - G_w)$ may never
be (in absolute value) large enough and further it may never have the same
sign over time, we shall assume $\varepsilon$ small and in a neighbourhood of unity to
account for the tendency of $\Sigma$ to move around its mean value in the long run.

\textsuperscript{15} See King and Robson (1993, p. 59) for details.
Conversely the value of $\delta$ will be assumed large enough (more than one) to give some meaningful effect of every given $\dot{G}_w$ on $\Sigma$. Since agents’ rational behaviour cannot normally generate extremely high value of $\dot{G}_w$ in continuous time, a higher value of $\delta$ account for short run shocks dealing with sudden changes of agents’ expectations.

- Relative changes of the balance of trade

$$
\Xi(G_f, G_n, G) = \zeta G_f + \sigma G_n - m - \mu G
$$

(12)

where $\zeta, \sigma, \mu > 0$ are sensitivity parameters of the balance of trade with respect to the foreign rate of growth, the technical progress and the domestic growth rate, respectively. The parameter $m > 0$ accounts for the assumption inasmuch as a constant domestic production without technical progress makes the balance of trade worse. This implies that we shall assume $\zeta G_f - m < 0$.

- The desired capital coefficient $C_r$

$$
\Phi(\gamma - \gamma_w) = C^* + \varphi(\gamma - \gamma_w)
$$

(13)

where $C^* > 1$ is an equilibrium value of the capital coefficient (i.e. a value such that $C_r = C$) and $\varphi > 1$ a reaction parameter which allows for the sensitivity of the firms to discrepancies between effective (actual) and expected relative changes of demand. The parameter $\varphi$ is assumed large enough to ensure meaningful changes in $C_r$. This is because $(\gamma - \gamma_w)$ will be relatively small (in absolute value) inasmuch as firms rationally adjust their expectations.

- The rate of growth of technical progress

$$
\dot{G}_n(\Sigma) = \beta(\xi - \Sigma)\Sigma
$$

(14)

The technical progress dynamics have been specified as a single peaked function to account for the analytical features we justified above. This is the simplest choice we might do about this function. The parameters $\beta > 1$ and $0 < \xi < 1$ reflect the level of the technical knowledge and are considered as
structural parameters. Our choice implies that the value of $\Sigma$ ensuring the maximum rate of growth of technical progress is $\bar{\Sigma} = \xi/2$, so that

$$G_w(\bar{\Sigma}) = \beta \xi^2/4 = \bar{G}.$$ 

Taking into account that differentiation of Eq. (8) yields

$$T \dot{G}_w + G_w = G \iff \dot{G}_w = \gamma(G - G_w)$$

(15)

where $\gamma = 1/T \geq 1$ is the speed of adjustment of the expected rate of growth to the actual one, if we put together equations from (10) to (15), after some rearrangement, we obtain the following non-linear four dimensional system:

$$\begin{align*}
\dot{G} &= \alpha \left\{ \left[ \phi + \psi \beta \left( \mu \xi + \delta \gamma \right) \right] G_w - \Sigma + x \right\} \\
\dot{G}_w &= \gamma(G - G_w) \\
\dot{\Sigma} &= \xi \left[ \beta(\xi - \Sigma) - G_w \right] + \delta \gamma(G - G_w) \\
\dot{x} &= \left[ \zeta G_f + \sigma \beta(\xi - \Sigma) - \mu G - m \right] x
\end{align*}$$

(H.1)

The system (H.1) has surely two fixed points if the balance of trade is in equilibrium\(^{16}\). These are: the origin, i.e. the stationary state of zero growth $P_0 = (0, 0, 0, 0)$, and a steady state solution $P^* = (G^*, \ G_w^*, \ \Sigma^*, \ 0)$ such that

$$G^* = G_w^* = \frac{\Sigma^*}{C^*}, \ \Sigma^* = \xi - \frac{1}{\beta C^*}, \ x^* = 0.$$ As $G^*$, $G_w^*$, $\Sigma^* > 0$ iff $\xi \beta C^* > 1$, we shall assume this last inequality always held. Furthermore, being $\bar{\Sigma} = \xi/2$ the saving rate ensuring the local maximum of $G_n$, we shall suppose $\Sigma^* \leq \bar{\Sigma}$ as the normal case when $x = 0$. The reason is that firms’ rational behaviour cannot imply an investment rate which yields a decreasing $G_n$ in the long run.

When $x \neq 0$ the “equilibrium” value of the growth rate $G (= G_w)$ arise from the fourth equation of the system (H.1), i.e. $\zeta G_f + \sigma G - \mu G - m = 0$, which implies

\(^{16}\) This case is completely equivalent to the one the closed economy yields. See Sportelli (2000) for details.
This last equation suggests that $\hat{G}$ may be positive iff $\sigma > \mu$, i.e. iff net exports are more sensitive to an increasing competitiveness than to the expansion of the domestic demand. Assuming that this is the case, it is important to point out here that the disequilibrium in the balance of trade makes the home growth rate entirely governed by elements concerned with the foreign trade. Therefore, if there is growth, it is an export-led growth.

Substitution of (16) in the third equation of the system (H.1) yields the quadratic equation

$$\beta(\xi - \Sigma)\Sigma - \hat{G} = 0$$

which has solutions

$$\Sigma = \frac{1}{2\beta} \left( \beta\xi \pm \sqrt{\beta^2 \xi^2 - 4\beta \beta \hat{G}} \right)$$

(17)

Since these solutions are complex for $\hat{G} > \beta\xi^2 / 4 = \max G_n = \bar{G}$, it follows that:

i) there is no additional fixed point for $\hat{G} > \bar{G}$;
ii) there is one additional fixed point for $\hat{G} = \bar{G}$;
iii) there are two additional fixed points for $\hat{G} < \bar{G}$.

The first case simply states that "$G_n$ sets a limit to the maximum average value of $G$ over a long period" (Harrod, 1948, p. 87). In fact, while $G$ may exceed $G_n$ for a short time only, the expected rate of growth $G_w$ may be greater than $G_n$ for a longer time. Consequently, as $G$ cannot stay greater than the increase of technological improvement allows, sooner than later it becomes lower than $G_n$, so that no equilibrium exists and the expected growth changes into a depression. This happens because increasing employment rates lead wages to overshoot the levels consistent with the average productivity dynamics and the induced inflation process makes a "vicious spiral depression" inevitable.
Even if by coincidence, when the second case holds, the export-led growth is such that the system may grow at the maximum level its technological improvements allow.

Finally, when the third case holds, the export-led growth generates two levels of the saving rate consistent with the steady state equilibrium of the economy. Let $\hat{\Sigma}_H > \hat{\Sigma}_L > 0$ be these two levels of the saving rate\(^{17}\). By substitution into the first equation of the system (H.1), we obtain the following equality:

$$C^* \hat{G} = \hat{\Sigma}_{H/L} - x$$ \hspace{1cm} (18)

which allows us to determine two levels of the net export rate (i.e. $\hat{x}_H$ and $\hat{x}_L$). Since both $C^*$ and $\hat{G}$ are given in the Eq. (18), the higher the value of the saving rate is, the higher the value of net export must be. Conversely, when the saving rate is at its lower level, net exports might be negative to yield the given rate of growth. In other words, the economic system would needs more imports than exports to sustain its growth.

It may be interesting to note at this point that since $\hat{G}$ yields two real values of the saving rate, we always have $\Sigma_L < \Sigma < \Sigma_H$ and consequently, $\hat{x}_L < \bar{x} < \hat{x}_H$ where $\bar{x}$ is the value net exports reach when the growth rate is at its maximum level. Therefore, looking at Harrod’s ingenious intuitions, we can infer that, as higher net exports tend to rise $G_w$ over $G_n$, the high level of the saving rate “is a force making for depression” (Harrod, 1948, p. 89). So that we should expect that the singular point $\hat{P}_H = (\hat{G}, \hat{G}_w, \hat{\Sigma}_H, \hat{x}_H)$ is a highly unstable fixed point. Equally unstable should be the singular point $\hat{P}_L = (\hat{G}, \hat{G}_w, \hat{\Sigma}_L, \hat{x}_L)$ because, if $\hat{x}_L < 0$ holds, whatever may be its level, net imports cannot be sustained indefinitely.

3.4 Qualitative analysis and numerical simulation (1st part)

To begin our qualitative analysis of the system (H.1), we have to consider its linearization in a neighbourhood of each critical point.

Relatively to the first two fixed points with $x = 0$, we obtain the following Jacobian matrices:

---

\(^{17}\) Descartes rule assures that these two levels of the saving rate are positive.
and

\[
\mathbf{J}^{(0)} = \begin{pmatrix}
0 & \alpha C^* & -\alpha & \alpha \\
\gamma & -\gamma & 0 & 0 \\
\delta \gamma & -(\varepsilon + \delta \gamma) & \varepsilon \beta \xi & 0 \\
0 & 0 & 0 & \zeta G_f - m
\end{pmatrix}
\]

A cofactor expansion of \((\mathbf{J}^{(0)} - \lambda \mathbf{I})\) and \((\mathbf{J}^{(*)} - \lambda \mathbf{I})\) yields the following characteristic polynomials:

\[
(\zeta G_f - m - \lambda)(\lambda^3 + a_0 \lambda^2 + b_0 \lambda + c_0) = 0,
\]

where \(a_0 = \gamma - \varepsilon \beta \xi\), \(b_0 = \gamma [\alpha (\delta - C^*) - \varepsilon \beta \xi]\), \(c_0 = \alpha \gamma \varepsilon (C^* \beta \xi - 1)\), and

\[
[\zeta G_f + (\sigma - \mu) G^* - m - \lambda](\lambda^3 + a_1 \lambda^2 + b_1 \lambda + c_1) = 0
\]

where \(a_1 = \gamma - \alpha \phi G^* - \varepsilon \beta (\xi - 2 \Sigma^*)\),

\(b_1 = \gamma \alpha (\delta - C^*) + (\alpha \phi G^* - \gamma) \varepsilon \beta (\xi - 2 \Sigma^*), \quad c_1 = \alpha \gamma \varepsilon [C^* \beta (\xi - 2 \Sigma^*) - 1].\)

Since one of the eigenvalues is surely real in both characteristic polynomials, we have to check if their cubic component has complex conjugate eigenvalues. It can be shown that a cubic equation has one real and two complex conjugate roots if the cubic discriminant \(D^3 > 0\). Let us note that this last inequality surely holds\(^\text{18}\) if \(a < 1\) and \(b > a\).

As far as the classification of these fixed points is concerned, we make use of the usual Routh-Hurwitz criterion. In what follows we shall impose \(\delta > C^*\).

\(^{18}\text{See Lorenz (1993, p. 106) for details.}\)
and \( \alpha(\delta - C^*) \geq \gamma \geq 1 \). Therefore, with reference to the second equilibrium point \( P^{(*)} \), we prove the following:

**Proposition 2:** Independently of the sign the real eigenvalue \( \lambda = \xi G_f + (\sigma - \mu) G^* - m \) may have, the equilibrium point \( P^{(*)} \) is a saddle-focus.

**Proof:** See Mathematical Appendix (section A.1).

![Graph](image)

*Figure 1: Limit cycle in two dimensions (x = 0)*

While the steady state solution with \( x = 0 \) is without remedy an unstable fixed point, closed orbits surrounding the stationary state of zero growth exist as it is shown in the following:

**Proposition 3:** Since the real eigenvalue \( \lambda = \xi G_f - m < 0 \), when \( c\beta \xi < \gamma \) and the parameter \( \varepsilon \) is assumed to be a critical parameter, according to the Routh-Hurwitz criterion, the cubic component of the characteristic equation may exhibit two complex conjugate eigenvalues with zero real part and no other real eigenvalue which equals zero iff \( a_0, b_0, c_0 \neq 0 \) and \( a_0b_0 - c_0 = 0 \). If a pair of purely imaginary eigenvalues and a non-zero real eigenvalue exists, then a Hopf bifurcation occurs in the system and a limit cycle emerges.

**Proof:** See Mathematical Appendix (section A.2).
Unlike the closed economy dynamics formalised by Sportelli (2000), in this extended version of the model there is only a limit cycle attracting all the system trajectories and no chaotic motion appears. In other words, this means that the opening to the foreign trade tends to stabilize the cycle, even if the growth path may initially wander through different points in the phase space before approaching the limit cycle. Figure 1 shows the result of the numerical simulation of our system in the plane \((G, \Sigma)\) under the assumption that only the two equilibria considered up to now exist. Figure 2 shows a three dimensional view of the limit cycle\(^{19}\).

![Figure 2: Limit cycle in three dimensions (x = 0)](image)

3.5 Qualitative analysis and numerical simulation (2nd part)

When we assume \(x \neq 0\), the system may exhibit either one additional fixed point where \(\hat{G} = \max G_n = \bar{G}\), or two others fixed points where \(\hat{G} < \bar{G}\).

These equilibria are added to the previous ones without changing their qualitative properties. So that, as the Jacobian matrices are the same, \(P(0)\) and \(P(\ast)\) remain unstable fixed points generating a limit cycle and a saddle focus respectively.

Looking at Eq. (16) it is easy to check that the new equilibria depend on the value the involved parameters and the foreign rate of growth may assume. Here we choose to consider as "critical" the parameter explaining the sensitivity of \(\dot{x}/x\) with respect to \(G_n\)^{20}, i.e. \(\sigma\). We interpret this parameter as the degree of competitiveness the economic system is able to display in the foreign markets through its \(G_n\).

---

\(^{19}\) All the simulations have been performed by using Mathematica 5.2 and the set of parameters specified in section 5.

\(^{20}\) Note that any other parameter involved in equation (16) might be chosen as critical.
Let $\sigma'$ be the value of $\sigma$ such that $\hat{G} = \bar{G}$. Since $\bar{\Sigma}$ is the value of the saving rate assuring $\max G_n$, the Jacobian matrix of the linearized system at the equilibrium point $P = (\bar{G}, \bar{G}_u, \bar{\Sigma}, \bar{x})$ is:

$$J_{|\sigma'=\sigma'} = \begin{pmatrix}
\alpha \phi \bar{G} & \alpha(C^* - \varphi \bar{G}) & -\alpha & \alpha \\
\gamma & -\gamma & 0 & 0 \\
\delta \gamma & -(\varepsilon + \delta \gamma) & 0 & 0 \\
-\mu \bar{x} & 0 & 0 & 0
\end{pmatrix}$$

where $\bar{x} = \frac{\bar{\xi}}{2} \left(1 - \frac{C^* \beta \bar{\xi}}{2}\right)$ is assumed to be positive and in a right-hand neighbourhood of zero.

The expansion of $(J_{|\sigma'=\sigma'} - \lambda I)$ gives rise to the characteristic equation

$$\lambda \left\{ \lambda^3 + a_2 \lambda^2 + b_2 \lambda + c_2 \right\} = 0$$

where $a_2 = \gamma - \alpha \varphi \bar{G}$, $b_2 = \alpha \left[\gamma (\delta - C^*) + \mu \bar{x}\right]$ and $c_2 = \alpha \gamma (\mu \bar{x} - \varepsilon)$. Since the determinant of $J_{|\sigma'=\sigma'}$ is zero, one of the eigenvalues of the characteristic polynomial is zero. Therefore $P$ is a non-hyperbolic fixed point.

As soon as $\sigma$ becomes more as $\sigma'$, two hyperbolic fixed points appear. The usual linearization of the system in the neighbourhood of these equilibria gives rise to the following Jacobian:

$$\tilde{J}_{|\sigma'=\sigma'} = \begin{pmatrix}
\alpha \varphi \hat{G} & \alpha(C^* - \varphi \hat{G}) & -\alpha & \alpha \\
\gamma & -\gamma & 0 & 0 \\
\delta \gamma & -(\varepsilon + \delta \gamma) & \epsilon \beta \left(\xi - 2\hat{\Sigma}_{H/L}\right) & 0 \\
-\mu \hat{x}_{H/L} & 0 & \epsilon \beta \left(\xi - 2\hat{\Sigma}_{H/L}\right) \hat{x}_{H/L} & 0
\end{pmatrix}$$
where the saving rate and net export are meant as functions of \( \sigma \) (i.e. \( \hat{\Sigma}_{H/L}(\sigma) \big|_{\sigma' = \sigma} \) and \( \hat{x}_{H/L}(\sigma) \big|_{\sigma' = \sigma} \)).

The expansion of \( \left( \frac{\hat{J}}{\sigma' - \lambda I} \right) \) allows us to define two 4th degree characteristic equations,

\[
\lambda^4 + a_{2\mu\nu} \lambda^3 + b_{2\mu\nu} \lambda^2 + c_{2\mu\nu} \lambda + d_{2\mu\nu} = 0,
\]

where:

\[
\begin{align*}
    a_{2\mu\nu} &= \gamma - \alpha \varphi \tilde{G} - \varepsilon \beta \left( \xi - 2\hat{\Sigma}_{H/L} \right), \\
    b_{2\mu\nu} &= \alpha \left[ \mu \hat{x}_{H/L} + \gamma \left( \delta - C^* \right) \right] - \left( \gamma - \alpha \varphi \tilde{G} \right) \varepsilon \beta \left( \xi - 2\hat{\Sigma}_{H/L} \right), \\
    c_{2\mu\nu} &= \alpha \left\{ \gamma \left( \mu \hat{x}_{H/L} - \varepsilon \right) + \left[ \gamma C^* \varepsilon - (\delta \gamma \sigma + \varepsilon \mu) \hat{x}_{H/L} \right] \beta \left( \xi - 2\hat{\Sigma}_{H/L} \right) \right\}, \\
    d_{2\mu\nu} &= \alpha \gamma \varepsilon \beta \left( \xi - 2\hat{\Sigma}_{H/L} \right) \hat{x}_{H/L} (\sigma - \mu).
\end{align*}
\]

With reference to all these alternative fixed points (i.e. either \( \bar{P} \) or \( \hat{P}_{H} \) and \( \hat{P}_{L} \) together), we prove the following:

**Proposition 4:** When \( \sigma \geq \sigma' \), if \( \xi \leq \varepsilon / \mu \) with \( \varepsilon > \bar{\varepsilon} \) and \( 0 < \gamma - \alpha \varphi \tilde{G} < 1 \), each new equilibrium point generated by the system has the same qualitative properties of \( \bar{P}^{(*)} \), i.e. each point is of a saddle-focus type.

**Proof:** See Mathematical Appendix (section A.4).

This analytical result is consistent with we said earlier about the equilibria \( \hat{P}_{H/L} \).

\( ^{21} \) The qualitative properties of these functions are described in the Mathematical Appendix (section A.3).
Therefore, according to Harrod, we can say that when \( G_w \) is pushed forward by a growing \( G \), whatever the cause may be, sooner or later \( G_w \) overcomes \( G_n \) whose growth never may evolve with the same quickness \( G \), and even more, \( G_w \) are growing at. The consequence is that sooner or later \( G \) ends up by getting behind \( G_w \) and the system will experiment the vicious spiral of depression. Simulations performed with \( \sigma > \sigma' \) show that every growth path starting near the equilibrium point \( \hat{P}_L \) screws around this point, then, either monotonically or spiralling, it will be pushed back toward the zero-growth point. When this region is reached, again it will start spiralling towards the limit cycle (see figures 3 and 4).

Instead, when the starting point is near \( \hat{P}_H \), the growth path wanders in the phase space and then approaches the limit cycle (see figure 5).

4. Final remarks

In this paper we offered an open economy macrodynamic model on Harrodian lines. The main purpose of the work is a reappraisal of the contemporary analysis of open economy macrodynamics based on the Keynes-Harrod-Kaldor tradition. After surveying the post-Keynesian approach to growth in open economy, we remarked that the influential Thirlwall’s contribution turned out to be unsatisfactory in its attempt at integrating two peculiar elements of Harrod’s analysis: foreign trade multiplier and dynamics. Therefore we followed a different procedure from Thirlwall’s one. We started...
from Harrod’s dynamics in closed economy and explored analytical implications stemming from the opening up of the economy to foreign trade. Particularly, we built on a recent reinterpretation of Harrod’s dynamics as suggested by Sportelli (2000) where business cycle and growth were jointly explained in a closed economy model. On the one hand, the extension of the model to open economy confirms some outcomes derived for closed economy dynamics; on the other hand some important differences emerge.

Figure 4: Limit cycle with $x \neq 0$, starting point near $\hat{P}_L$ and $\sigma = 3.9$

As regards similarities, open and closed economy dynamics share the following common peculiarities:

i) Both economies need negative saving rates near the height of a recession (this happens because there is no autonomous investment in the model), but as Harrod himself wrote “the sooner this happens, the sooner will the slump be arrested (Harrod, 1996, p. 266).

ii) The maximum rate of advance or recession occurs at the moment when $G_w$ equals $G$. This confirms that the equilibrium between expected (warranted) and effective (actual) growth has nothing to do with the equilibrium of the economic system; a simple 45° degree line drawn in the plane ($G$, $G_w$) is able to visualize this result (see Fig. 6).

iii) The steady state growth, in its common meaning, is highly unstable. This is true in any case (i.e. either the economy is closed or opened to foreign trade). So that, according to Harrod we could say that “there is, in the real world, no steady advance” (Harrod, 1948, p. 59).

iv) Whether regular or irregular cycles may be, they are asymmetric with respect to the origin; the consequence is that the mean value of $G$ will be positive in the long run, because trajectories are stretched towards the region where $G$ has positive values. This result should lead us to think that long run

---

Note that in the closed economy only one steady state exists, while, in the opened economy there might be until two steady states growth (one with $x = 0$ and one or two with $x \neq 0$).
positive growth rates observed in the real world might be consistent with stylized facts our model is able to describe, if these growth rates were interpreted as an average of different rates of growth with different signs coming from different phases of the cycle the economy experiments over time\textsuperscript{23}.

Figure 5: Limit cycle with $x \neq 0$, starting point near $\hat{P}_H$ and $\sigma = 3.9$

Nevertheless, with respects to the following aspects open economy dynamics departs from the one relative to the closed economy:

i) While chaotic instability epitomizes a closed economy (as shown in Sportelli, 2000), dynamics shift to a limit cycle in open economy. This formal result confirms Harrod’s intuition of a more moderate cyclical instability emerging in open economy.

ii) The open economy growth generates two levels of the saving rate consistent with the steady state equilibrium. When the saving rate $\Sigma$ is determined at its higher level, usual export-led growth is generated (because the higher the value of $\Sigma$ is, the higher the value of $x$ must be); but when $\Sigma$ is at its lower level, net exports might be negative in order to sustain a given rate of growth. In this case “import-led” growth is generated.

This last result disagrees with the Dixon-Thirlwall export-led growth model, where the link between export, productivity and growth is circular and cumulative and so, exports remain the main engine of growth also when balance of payments constraint is introduced explicitly in the analysis (Thirlwall, 1978). In contrast, our model shows that export-led growth dynamics is one of the possible outcomes deriving from the endogenous interaction between technical progress and capital accumulation. On this

\textsuperscript{23} The set of parameters here used for the simulations yields an average value of $G$ included between 0.21% and 0.27%. Instead, the maximum value $G$ may reach during a growth phase is about 2.02%.
respect, our model agrees with one of the main results of the equilibrium approach to macrodynamics, where random shocks due to technological changes are one of the causes of the business cycle. Although our model does not require random shocks to generate fluctuations in the economic activity, technological progress still remains the main impulse propagation mechanism able to drive and destabilise the growth process.

5. Parameters

\[ \alpha = 0.5 \quad C^* = 4 \quad \varphi = 15 \quad \gamma = 1 \]
\[ \varepsilon \text{ from } 0.2 \text{ to } 1.079 \quad \beta = 2.5 \quad \xi = 0.18 \quad \delta = 6.2 \]
\[ 2 \leq \sigma < 4 \quad m = 0.07 \quad \mu = 1.4 \quad \zeta = 1.9 \]
\[ G_f = 0.03. \]
\[ \max G_n = \bar{G} = 2.025 \quad \Sigma = 0.9 \]

Figure 6: Multiple equilibria \( G = G_w \)
A.1 Proof of proposition 2

As we assumed $\Sigma^* \leq \xi = \xi/2$, it follows that $\xi - 2\Sigma^* \geq 0$. So that if $0 < a_i < 1$, i.e. $1 > \gamma - \alpha \varphi G^* > \varepsilon \beta (\xi - 2\Sigma^*)$, it will result $b_i > a_i > 0$ because we assumed $\alpha (\delta - C^*) \geq \gamma > 1$. Therefore, the cubic component of the characteristic polynomial exhibits a positive cubic discriminant and consequently possesses one real and two complex conjugate roots.

Furthermore, as we set, it cannot be $c_1 > 0$ because

$$C^* \beta (\xi - 2\Sigma^*) - 1 > 0 \Rightarrow C^* \beta \left[ \xi - 2 \left( \frac{1}{C^* \beta} \right) \right] - 1 > 0 \Rightarrow \xi C^* \beta < 1. $$

So that, given $c_1 < 0$, the cubic component of the characteristic polynomial has one real root $\lambda_i > 0$. By setting as $\rho_i \pm \omega_i$ the two complex conjugate roots, according to Vieta’s formula we can write:

$$a_i b_i - c_i = -\left[ \lambda_i + (\rho_i + \omega_i) \right] \left[ \lambda_i + (\rho_i - \omega_i) \right] \left[ (\rho_i + \omega_i) + (\rho_i - \omega_i) \right] > 0. $$

Thus, after some algebraic rearrangement, it follows that $\rho_i < 0$. It then turns out that $P^{(i)}$ is a locally unstable equilibrium. Specifically, since the complex eigenvalues have negative real part and, at least one of the two real eigenvalues is positive, $P^{(i)}$ is an equilibrium of a saddle-focus type (Glendinning, 1994, pp. 351-358).

Note that if $\gamma = 1$, it will surely be $0 < a_i < 1$ and $b_i > a_i$. This implies that the cubic discriminant is still positive.
A.2 Proof of proposition 3

As \( m > \zeta G_j \) and \( c_0 = \alpha \gamma \epsilon (C^* \beta \xi - 1) > 0 \), because \( C^* \beta \xi > 1 \), the characteristic polynomial of the Jacobian matrix \( J^{(0)} \) will definitely have two negative real eigenvalues.

Looking at the cubic component of the characteristic polynomial, we can infer that \( \epsilon \beta \xi < 1 \) implies \( 0 < a_0 < 1 \), if \( \gamma \) is enough near 1, and further that \( b_0 \geq a_0 \) because of \( \alpha (\delta - C^*) \geq \gamma \). Therefore, that cubic component has a positive discriminant, and then it gives rise to a pair of complex conjugate roots whose real part will have a sign depending on the following inequality:

\[ a_0 b_0 - c_0 \geq 0 \]

As \( \epsilon \) is assumed as a critical parameter, given the specific values of \( a_0, b_0 \) and \( c_0 \), we can write:

\[
a_0 b_0 - c_0 = (\gamma - \epsilon \beta \xi) \cdot \gamma \left[ \alpha (\delta - C^*) - \epsilon \beta \xi \right] - \alpha \gamma \epsilon (C^* \beta \xi - 1) \geq 0 \quad (A.1)
\]

Let \( \epsilon' < \gamma / \beta \xi \) be an initial value of \( \epsilon \) such that the inequality (A.1) is positive (so that, according to the Routh-Hurwitz conditions, \( P_0 \) is a stable fixed point). If \( \epsilon \) increases, it results \( \partial a_0 / \partial \epsilon < 0 \) and \( \partial c_0 / \partial \epsilon > 0 \); as a consequence the product \( a_0 b_0 \) will be equal zero as soon as \( \epsilon = \gamma / \beta \xi \). It follows that \( a_0 b_0 - c_0 = -c_0 < 0 \). This implies that there exists a value \( \epsilon' < \overline{\epsilon} < \gamma / \beta \xi \) such that \( a_0 b_0 - c_0 = 0 \) and \( a_0 > 0 \). Therefore, the characteristic polynomial will have a pair of complex conjugate roots \( \rho_0 \pm \alpha i \) with negative/positive real part (\( \rho_0 \leq 0 \)) if \( \epsilon \leq \overline{\epsilon} \). Given that a pair of purely imaginary eigenvalues exists when \( \epsilon = \overline{\epsilon} \) (so that (A.1) holds as an equality) and the other two real eigenvalues are negative, the Hopf bifurcation theorem allows us to state that the system (H.1) loses its stability and displays closed orbits in a neighbourhood of the zero growth equilibrium \( \overline{\epsilon} \) when \( \epsilon > \overline{\epsilon} \).

---

25 Note that \( \partial b_0 / \partial \epsilon < 0 \), but \( \alpha (\delta - C^*) > \gamma \) assures that \( a_0 = 0 \) when \( b_0 \) is still positive.

26 We have to notice that the expression (A.1) is quadratic in \( \epsilon \). This implies that two bifurcation values (both positive) exist. Here we are interested only to the first one because the second bifurcation value of \( \epsilon \) is too high to be consistent with our assumptions. Our parameters yield the following bifurcation values for \( \epsilon \): \( \epsilon_1 \cong 0.4776 \) and \( \epsilon_2 \cong 2.8434 \).
A.3 Qualitative properties of the functions \( \hat{\Sigma}_{H/L}(\sigma) \) and \( \hat{x}_{H/L}(\sigma) \)

Taking into account eq. (10), these functions have the following specific form:

\[
\hat{\Sigma}_{H/L}(\sigma) \bigg|_{\sigma > \sigma'} = \frac{\xi}{2} \pm \frac{\beta(\bar{G} - \hat{G}(\sigma))}{2\beta} \; ;
\]

\[
\hat{x}_{H/L}(\sigma) \bigg|_{\sigma > \sigma'} = \frac{\xi}{2} \pm \frac{\sqrt{\beta(\bar{G} - \hat{G}(\sigma))}}{2\beta} - C'\hat{G}(\sigma) .
\]

Let us note that \( \sigma \to \sigma' + \) implies \( \hat{\Sigma}_{H/L} \to \bar{\Sigma} = \frac{\xi}{2} \) and \( \hat{x}_{H/L} \to \bar{x} \), because

\[
\lim_{\sigma \to \sigma'^+} \hat{G}(\sigma) = \bar{G} = \max G_n .
\]

Furthermore we have:

\[
\frac{\partial \hat{\Sigma}_H}{\partial \sigma} \bigg|_{\sigma > \sigma'} = -\frac{\partial \hat{G}/\partial \sigma}{\sqrt{\beta(\bar{G} - \hat{G}(\sigma))}} > 0 \quad \text{and} \quad \frac{\partial \hat{\Sigma}_L}{\partial \sigma} \bigg|_{\sigma > \sigma'} = -\frac{\partial \hat{G}/\partial \sigma}{\sqrt{\beta(\bar{G} - \hat{G}(\sigma))}} < 0 ,
\]

because \( \frac{\partial \hat{G}}{\partial \sigma} < 0 \); as a consequence, \( \frac{\partial \hat{x}_H}{\partial \sigma} \bigg|_{\sigma > \sigma'} = \frac{\partial \hat{\Sigma}_H}{\partial \sigma} - C' \frac{\partial \hat{G}}{\partial \sigma} > 0 \) while

\[
\frac{\partial \hat{x}_L}{\partial \sigma} \bigg|_{\sigma > \sigma'} = \frac{\partial \hat{\Sigma}_L}{\partial \sigma} - C' \frac{\partial \hat{G}}{\partial \sigma} \leq 0 .
\]

When \( \frac{\partial \hat{x}_L}{\partial \sigma} \bigg|_{\sigma > \sigma'} = 0 \), it can be easily shown that the function \( \hat{x}_L \) is at a local minimum inasmuch as at that point \( \frac{\partial^2 \hat{x}_L}{\partial \sigma^2} \bigg|_{\sigma > \sigma'} > 0 \).

Finally, although unrealistic, if \( \sigma \to \infty \), it results:

\[
\lim_{\sigma \to \infty} \hat{\Sigma}_H = \hat{x}_H = \xi \quad \text{and} \quad \lim_{\sigma \to \infty} \hat{\Sigma}_L = \hat{x}_L = 0 .
\]

This because \( \lim_{\sigma \to \infty} \hat{G}(\sigma) = 0 . \)
The assumption that $\sigma$ may assume any value in the interval $]\sigma', \infty[$ is unacceptable from the economic point of view. Higher values of $\sigma$ are such that to give $\hat{G}(\sigma) \leq 0$; as a consequence, low rates of growth of income and technical progress become inconsistent with a higher degree of competitiveness on the foreign markets. This led us to consider $\sigma$ being upper bounded in our model. The set of $\sigma$ values we took in the model (from section 3.5 on) has been therefore $[\sigma', 4[$. Figure A.1 shows the functions $\hat{\Sigma}_{III}$ and $\hat{x}_{III}$ plotted on that domain by using the set of parameters given above in section 5.
A. 4 Proof of proposition 4

Given \( \sigma > \sigma' \), according to the Routh-Hurvitz criterion, the equilibria \( \hat{P}_{H/L} \) will be locally stable if the following inequalities are jointly fulfilled:
\[
\begin{align*}
&\text{and } a_{2_{H/L}} > 0 \text{ and } a_{2_{H/L}}\left(b_{2_{H/L}} + c_{2_{H/L}} - a_{2_{H/L}}\cdot d_{2_{H/L}}\right) - c_{2_{H/L}}^2 > 0. \\
&\text{and } (b_{2_{H/L}} + c_{2_{H/L}} - a_{2_{H/L}}\cdot d_{2_{H/L}}) - c_{2_{H/L}}^2 > 0.
\end{align*}
\]

Let us note that since \( \hat{\Sigma}_L < \Sigma < \hat{\Sigma}_H \), it will surely be \( \xi - 2\hat{\Sigma}_H < 0 \) and \( \xi - 2\hat{\Sigma}_L > 0 \). Taking into account that \( \gamma(\delta - C^*) > 1 \) and given that \( \hat{G} < \overline{G} \), there is no difficulty to check that \( a_{2_{H/L}} > 0 \), \( b_{2_{H/L}} > 0 \) and \( d_{2_{H/L}} < 0 \) if \( \hat{x}_L < 0 \). If \( \hat{x}_L > 0 \), and this is possible only if \( \hat{x}_L \) is close

\[
\begin{align*}
\hat{\Sigma}_H & \quad \hat{\Sigma}_L \\
\hat{x}_H & \quad \hat{x}_L
\end{align*}
\]

\textit{Figure A.1: Saving rate and net export as functions of competitiveness on the foreign market.}

enough to \( \overline{x} \) (which is near zero), then \( d_{2_{H/L}} > 0 \) and in a neighbourhood of zero because \( \hat{\Sigma}_L \square \xi/2 \).
As far as the sign of \( c_{2_HL} \) are concerned, some ambiguity rises inasmuch as these coefficients directly depend on the value of \( \sigma \). Therefore, if \( \sigma \rightarrow \sigma' + \), \( c_{2_HL} \rightarrow \alpha \gamma \left( \mu \bar{x} - \varepsilon \right) < 0 \), because the term 
\[
\left[ \gamma C^* \varepsilon - \left( \delta \gamma \sigma + \varepsilon \mu \right) \hat{x}_{H/L} \right] \beta \left( \bar{\xi} - 2 \hat{\Sigma}_{H/L} \right) \to 0,
\]
and the term 
\[
H L c x \alpha \gamma \mu \varepsilon \rightarrow -
\]
\[\sigma \to \sigma +', \]
\[
(\bar{\xi} - 2 \hat{\Sigma}_{H/L})
\]
tends to vanish. On the contrary, if \( \sigma \) is high enough, this term becomes positive and dominant with respect to 
\[
\gamma \left( \mu \hat{x}_{H/L} - \varepsilon \right),
\]
doing that \( c_{2_HL} > 0 \). A numerical study of the expression defining 
\[
c_{2_H} \text{ and } c_{2_L} \text{ shows that these coefficients reach respectively a neighbourhood of zero (from the left-hand side) when } 8.54 < \sigma < 8.55 \text{ and } 2.28 < \sigma < 2.29.
\]
Given the upper bound of \( \sigma \) considered in the former section A.3, it then turns out that \( c_{2_H} < 0 \) and \( c_{2_L} \approx 0 \).

All this allow us to state that both \( \hat{P}_H \) and \( \hat{P}_L \) are locally unstable fixed points. Nevertheless, if \( \hat{P}_H \) needs no particular comment, it might be useful to point out that \( \hat{P}_L \) may never be stable because, independently of the sign the condition 
\[
a_{2_HL} \left( b_{2_HL} \cdot c_{2_HL} - a_{2_HL} \cdot d_{2_HL} \right) - c_{2_HL}^2 \text{ exhibits, if } d_{2_L} > 0, \text{ it is } \hat{x}_L \not\subseteq \bar{x}
\]
which implies \( \sigma \) close to \( \sigma' \) and therefore \( c_{2_L} < 0 \); vice versa, if \( \sigma \) is far enough from \( \sigma' \) and such that to yield \( c_{2_L} > 0 \), it will be \( \hat{x}_L < 0 \) and consequently \( d_{2_L} < 0 \).

As is well known, the Routh-Hurwitz criterion does not provide an answer to the question whether the eigenvalues are real or complex. Thus to investigate the nature of the roots of the quartic characteristic polynomial we had resort to some algebraic tools.

Let \( f_{H/L}(\lambda) \) be the quartic polynomial obtained by the expansion of 
\[
\begin{pmatrix}
\mathbf{j} & -\lambda \mathbf{I}
\end{pmatrix}
\]
As usual, by setting \( \lambda = y - \frac{a_{2_HL}}{4} \), we find the following reduced form,
\[
f_{H/L}(y) = y^4 + p_{H} y^3 + q_{H_L} y^2 + r_{H_L} y + s_{H_L} \tag{A.1}
\]
where:
The discriminant of eq. (A.1) is given by
\[ D = -4p_{n|L} q_{n|L} r_{n|L} + 27 p_{n|L}^2 q_{n|L}^2 + 16 p_{n|L} r_{n|L} - 128 p_{n|L}^2 r_{n|L}^2 + 256 r_{n|L}^3. \]

As we know from algebra, the quartic function (A.1) – and consequently \( f_{n|L}(\lambda) \) – will have two real and two complex conjugate roots if \( D < 0 \). The roots will be all real and distinct if \( D > 0 \) and, simultaneously, \( p_{n|L} < 0 \) and \( p_{n|L}^2 - 4q_{n|L} r_{n|L} > 0 \). If these last inequalities are not jointly satisfied, then the four roots will be two by two complex and conjugate.

The former assumptions on our coefficients give \( p_{n|L} > 0 \) and are such that to yield \( p_{n|L} \) and \( |q_{n|L}| \) both more than \( r_{n|L} \) which is in a neighbourhood of zero. In this case \( D < 0 \) and \( \hat{J}_{\lambda} \) will have two real and two complex eigenvalues. This result is supported by a wide range of parameters preserving all our main economic hypotheses.
As far as the signs of the real eigenvalues are concerned, we have to look at the sign of $d_{2\eta L}$. Given that $d_{2\eta L} < 0$, by virtue of Vieta’s formula we can infer that the real eigenvalues have opposite sign.

Let $\lambda_{1\eta L} > 0$ and $\lambda_{2\eta L} < 0$ be these two real eigenvalues. If $\rho_{2\eta L} \pm i\omega_{2\eta L}$ are the complex conjugate roots of $f_{\eta L}(\lambda)$, Vieta’s formula allow us to set $a_{2\eta L} = 2\delta_{2\eta L} + \lambda_{1\eta L} + \lambda_{2\eta L} > 0$ and, if $\sigma$ is close enough to $\sigma'$, $c_{2\eta L} = -\left[\left(\rho_{2\eta L}^2 + \omega_{2\eta L}^2\right)\left(\lambda_{1\eta L} + \lambda_{2\eta L}\right) + 2\rho_{2\eta L}\lambda_{1\eta L}\lambda_{2\eta L}\right] < 0$.

If it were $\rho_{2\eta L} > 0$ and $\lambda_{1\eta L} + \lambda_{2\eta L} < 0$ $\Leftrightarrow$ $\lambda_{1\eta L} < |\lambda_{2\eta L}|$, $a_{2\eta L} > 0$ and $c_{2\eta L} < 0$ might not be simultaneously satisfied. Similarly, the assumption $\rho_{2\eta L} > 0$ and $\lambda_{1\eta L} + \lambda_{2\eta L} > 0$ $\Leftrightarrow$ $\lambda_{1\eta L} > |\lambda_{2\eta L}|$ might not be true because $a_{2\eta L} > 0$ $\Rightarrow$ $\lambda_{1\eta L} + \lambda_{2\eta L} > -2\rho_{2\eta L}$ and since $c_{2\eta L} < 0$ $\Rightarrow$ $-\left(\rho_{2\eta L}^2 + \omega_{2\eta L}^2\right)\left(\lambda_{1\eta L} + \lambda_{2\eta L}\right) - 2\rho_{2\eta L}\lambda_{1\eta L}\lambda_{2\eta L} < 0$, straightforwardly it should be

$\left(\rho_{2\eta L}^2 + \omega_{2\eta L}^2\right)2\rho_{2\eta L} - 2\rho_{2\eta L}\lambda_{1\eta L}\lambda_{2\eta L} < 0$ $\Leftrightarrow$ $\left(\rho_{2\eta L}^2 + \omega_{2\eta L}^2\right) < \lambda_{1\eta L}\lambda_{2\eta L}$ which is impossible because $\lambda_{1\eta L}\lambda_{2\eta L} < 0$. Conversely, if $\rho_{2\eta L} < 0$, to have simultaneously $a_{2\eta L} > 0$ and $c_{2\eta L} < 0$, it must be $\lambda_{1\eta L} + \lambda_{2\eta L} > 0$, because $\lambda_{1\eta L} + \lambda_{2\eta L} < 0$ should imply $a_{2\eta L} < 0$.

All this allow us to state that the equilibria $\hat{P}_{\eta L}$ are of a saddle-focus type inasmuch as $\rho_{2\eta L} < 0$, $\lambda_{1\eta L} > 0$ and $\lambda_{2\eta L} < 0$. The numerical study and simulations confirm that this qualitative property of $\hat{P}_{\eta L}$ is preserved for a wide range of parameters consistent with all our assumptions.

Finally, if we consider $\sigma = \sigma'$, the characteristic polynomial becomes

$$f(\lambda) = \lambda \left\{ \lambda^3 + a_2 \lambda^2 + b_2 \lambda + c_2 \right\} = 0.$$
one done in section A.1. This confirm that the equilibrium point $\bar{P}$ is still of a saddle-focus type.
References


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<td>2002</td>
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<tr>
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